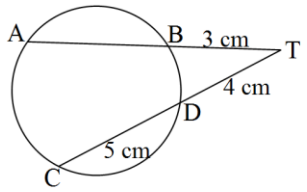
MASENO SCHOOL

Topic 10: CIRCLES: Chords and Tangents

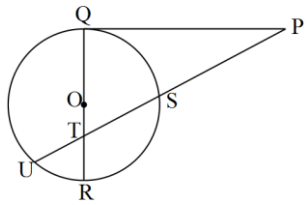
By Mr. Patrick Mboya

Intersecting Chords

1. In the figure below, the chords CD and AB intersect externally at T. DT = 4 cm, BT = 3 cm and CD = 5 cm, calculate the length AB.



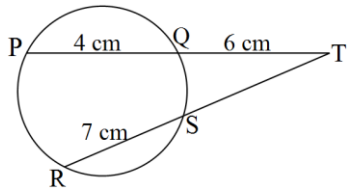
2. In the figure below, PQ is a tangent to the circle at point Q. PQ meets chord SU produced at point P. QR is a diameter of the circle. O is the centre of the circle. QR meets SU at point T.



(i) Given that QP = 15cm and UP = 25cm. Calculate the length of line SU.

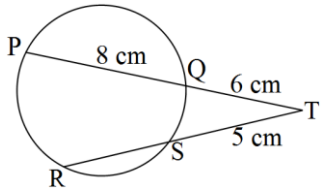
(ii) If RT = 5cm and ST: TU = 5:3. Calculate the length of line QT

3. In the figure below, chords PQ and RS intersect externally at T.

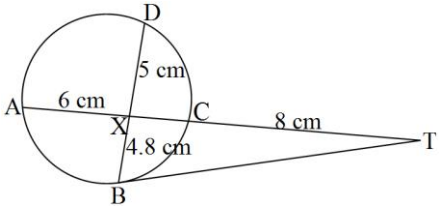


Given that PQ = 4cm, QT = 6cm and RS = 7cm, find RT.

4. Two chords PQ and RS are produced to meet at T. Given that PQ = 8cm, QT = 6cm and ST = 5cm. Find the length RS.



5. In the figure below, BT is a tangent to the circle at B. AXCT and BXD are straight lines. AX = 6 cm, CT = 8 cm, BX = 4.8 cm and XD = 5cm (*figure not drawn to scale*)



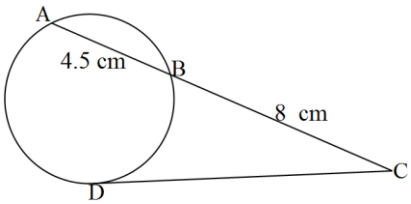
Find the length of:

(a) XC

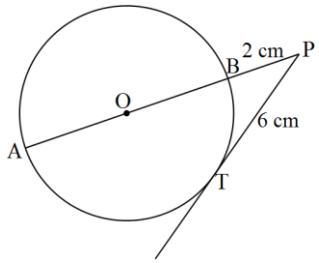
(b) BT

6. Two chord AB and CD intersect internally at X. AX = 10 cm, XB = 8 cm and CD = 21 cm. Calculate the length of XD.

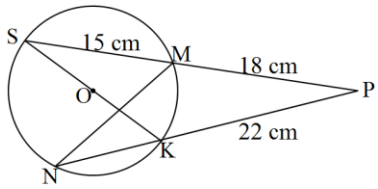
7. In the figure below the tangent DC meets the chord AB produced at C. AB = 4.5 cm and BC = 8 cm. Calculate the length of DC.



8. In the given figure below, O is the centre of the circle and AOBP is a straight line. PT is a tangent to the circle at T. If PT = 6 cm and BP = 2 cm, find the radius of the circle.



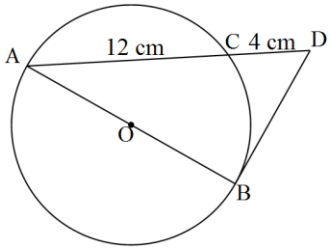
9. The figure below shows external intersection of two chords SM and NK which are produced to meet at P. Given that SOK is the diameter of the circle and that SM = 15 cm, MP = 18 cm and PK = 22 cm.



(a) Calculate the length of chord NK.

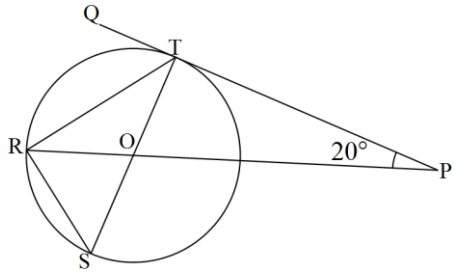
(b) Calculate the radius of the circle to 1 decimal place.

10. In the figure below, AOB is a diameter to the circle centre O and BD is a tangent to the circle at B. $AC = 12cm$ and $CD = 4cm$. Giving your answer in its simplest form, calculate the exact radius of the circle.



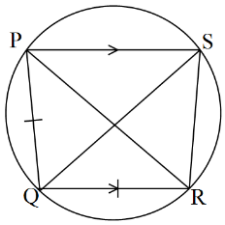
Angle Properties of a Circle

1. In the figure below, R, T and S are points on the circle centre O. PQ is a tangent to the circle at T. POR is a straight line and $\angle QPR = 20^0$.



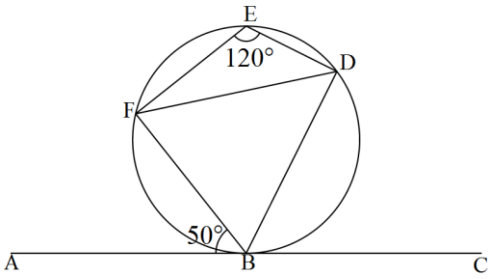
Find the size of angle RST.

2. In the figure below, PQRS is a cyclic quadrilateral PQ = QR, angle PQR = 104^0 and PS is parallel to QR.

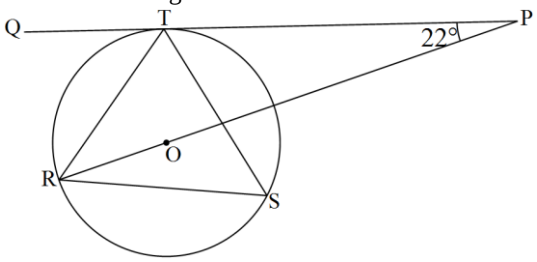


Determine the size of:
(a) Angle PSR
(b) Angle PQS

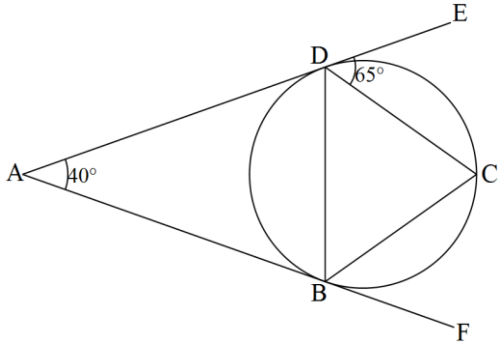
3. Find angles FBD, FDB and DBC given that ABC is tangent to the circle at B and $\angle ABF = 50°$, $\angle FED = 120^0$.



4. In the figure below, R, T and S are points on a circle centre O. PQ is a tangent to the circle at T. POR is a straight line and angle QPR = 22°. Find the size of Angle RST.

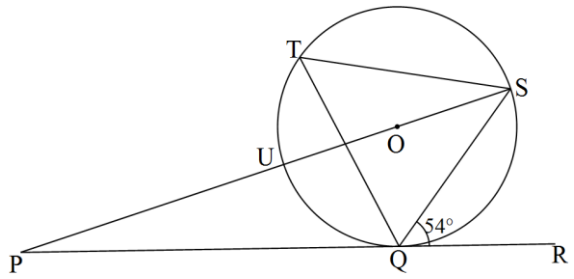


5. In the figure below, ADE and ABF are tangents to the circle at D and B respectively. $\angle DAB = 40^o$ and $\angle CDE = 65^o$.

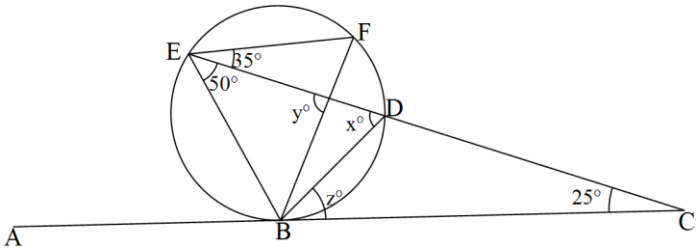


Giving a reason for your answers, find the size of:
(a) $\angle ADB$
(b) $\angle ABC$

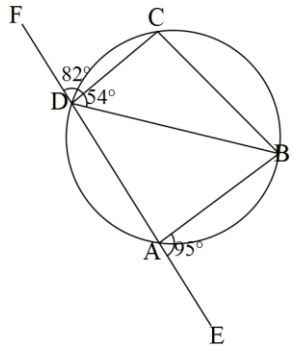
6. In the figure below, SOUP is a straight line that passes through the centre O of the circle. PQR is a tangent to the circle at Q and $\angle SQR = 54^o$. Calculate the size of angle UPQ.



7. In the figure below, ABC is a tangent to the circle at B and CDE is a straight line. $\angle BED = 50^o$, $\angle DEF = 35^o$ and $\angle ECB = 25^o$. Calculate the angles marked x, y and z.



8. In the figure below, EADF is a straight line, $\angle CDF = 82^o$, $\angle BDC = 54^o$ and $\angle BAE = 95^o$.

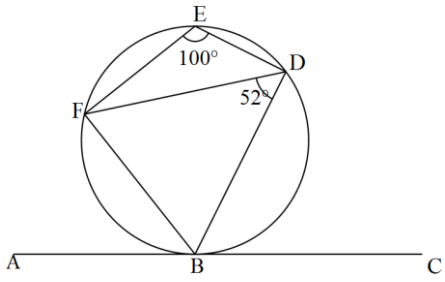


Calculate the size of:

(a) $\angle ABD$

(b) $\angle CBD$

9. In the figure below, ABC is tangent to the circle at B and $\angle BDF = 52°$, $\angle FED = 100^0$.

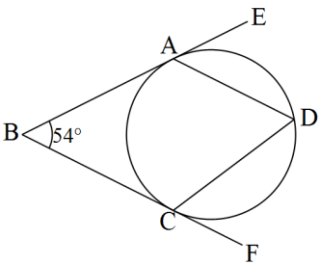


Calculate the size of:

(a) $\angle FBA$

(b) $\angle DBC$

10. In the figure below, BAE and BCF are tangents to the circle at A and C respectively. $\angle ABC = 54^o$. Calculate the size of $\angle ADC$.



Construction

1. Using a ruler and a pair of compasses only, construct a triangle LMN in which LM = 5 cm, LN = 4.2 cm and angle MLN = 45^0. Draw the circle that touches all the sides of the triangle.

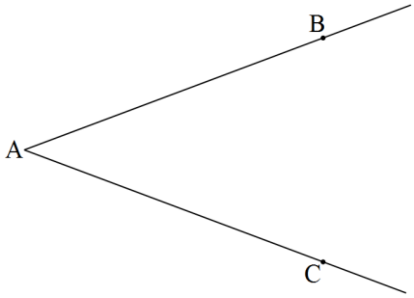
2. Triangle ABC is such that AB = 5.5 cm, BC = 4.6 cm and AC = 3.6 cm. By construction, locate the centroid, D, of the triangle. Measure AD.

3. Using a pair of compasses and a ruler only, construct a triangle XYZ in which XY = 2.5 cm, XZ = 2.1 cm and $\angle ZXY = 45^o$. Escribe a circle to touch side ZY of triangle XYZ drawn. Measure the radius of the circle.

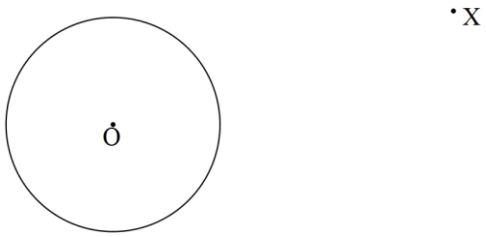
4. Draw a triangle PQR with PQ = 4.5 cm, QR = 3.5 cm and RP = 3.2 cm. Locate the orthocenter C and measure CQ.

5. Using a ruler and a pair of compasses only, draw a circle center O and radius 2.5 cm. Construct a tangent 6 cm long from a point P outside the circle to Q. Measure PQ.

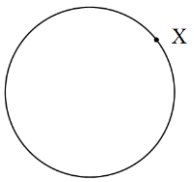
6. AB and AC are tangents to a circle at B and C. Construct the circle and hence measure its radius.



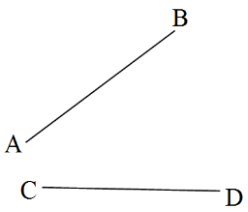
7. Using a ruler and a pair of compasses only, draw two tangents to the circle centre O at points P and Q from the external point X. Measure the length of the tangents.



8. A point X is on a circle below. By construction, locate the centre of the circle hence draw a tangent 6 cm long to the circle at X.



9. AB and CD are tangents to a circle centre O. Construct the circle and measure its radius.

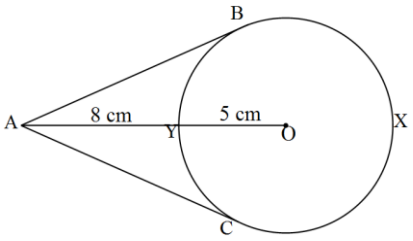


10. An ex – circle to a triangle ABC touches the side BC. The radius of the ex – circle is 3.5 cm and $\angle CAB = 60^o$. By construction, draw the ex – circle and measure the side BC of the triangle.

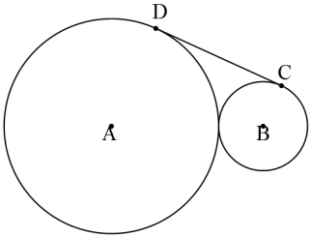
General Exercises

1. A chord is 5 cm from the centre of a circle whose radius is 13 cm. Calculate the length of the chord.

2. In the figure below, AB and AC are tangents to the circle at B and C respectively. AY = 8 cm and the radius of the circle is 5 cm. Calculate the length of the major arc BXC to 2 d.p. Take $\pi = 3.142$.

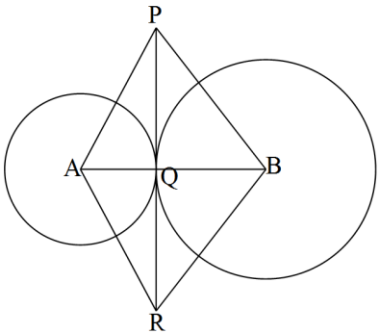


3. In the figure below, DC is a common tangent to the circles centres A and B. The radius of circle centre A is 9 cm and the radius of the circle centre B is 4 cm. Calculate the length of the tangent DC.

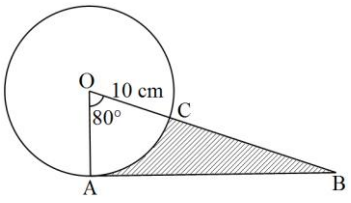


4. Three sides of a triangle AB, BC and AC are tangents to a circle centre O at D, E and F respectively. Given that AB = 11 cm, BC = 10 cm and AC = 9 cm, find the length FC.

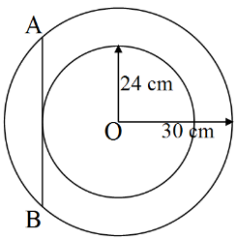
5. In the figure below, PQR is a common tangent to the circles centre A and B. $AP = 13cm$, $AB = 14cm$ and $BP = 15cm$. Calculate the length of the chord PR if $PB = BR$.



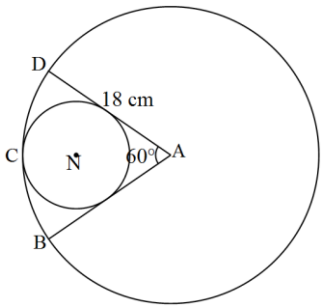
6. In the figure alongside, AB is a tangent to the circle centre O and radius 10 cm. The arc AC subtends an angle of 80^0 at the centre of the circle. Calculate the area of the shaded region correct to four significant figures. Take $\pi = 3.142$.



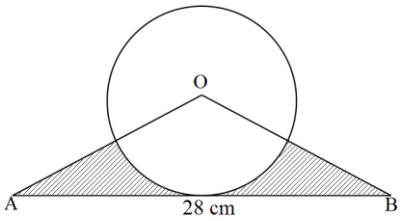
7. The figure below shows two concentric circles with centres at O and radii 24 cm and 30 cm. AB is a tangent to the smaller circle. Calculate the length of AB.



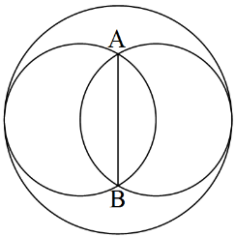
8. In the figure below the circle centre A and N touches internally at C and Angle BAD = 60°. If the radius of the circle A is 18 cm. Find the radius of circle N.



9. The figure below shows a circle centre O. The line AB = 28 cm is a tangent to the circle such that OA = OB and Angle AOB = 120°. Calculate the area of the shaded region. Take $\pi = 3.142$.

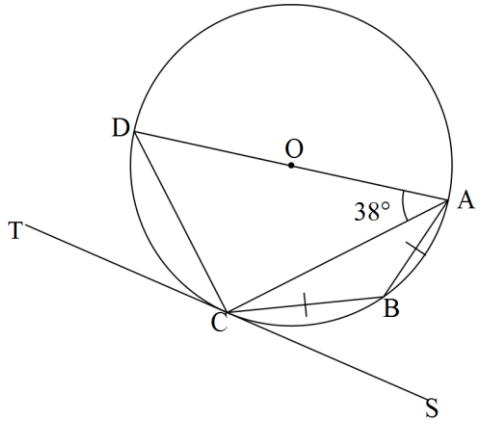


10. In the figure below, two small circles of radius 3 cm each are drawn inside a larger circle of radius 4 cm. The small circles intersect at A and B. Calculate the length of AB.



Section II Questions

1. In the figure below, AD is a diameter of the circle ABCD centre O, radius 10 cm. TCS is a tangent to the circle at C. AB = BC and Angle DAC = 38°.



(a) Find the size of angle:

(i) ACS (2 marks)

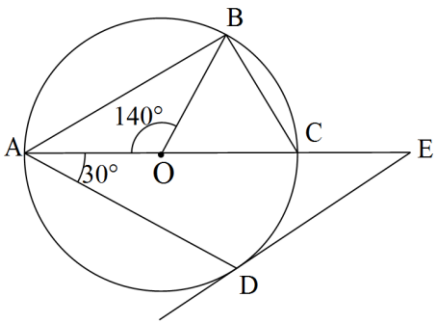
(ii) BCA (2 marks)

(b) Calculate, to 2 d.p, the length of:

(i) AC (3 marks)

(ii) AB (3 marks)

2. The figure below A, B and D are points on the circumference of a circle centre O. The points A, O, C and E are on straight line and ED is a tangent to the circle at D. Angle AOB = 140^0 and angle CAD = 30^0.



Giving reasons, find:

(a) ∠CBD (2 marks)

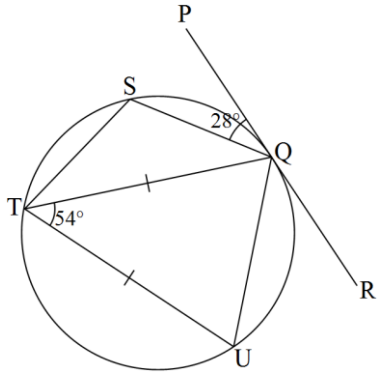
(b) ∠OBD (2 marks)

(c) ∠BDE (2 marks)

(d) ∠CED (2 marks)

(e) ∠BCD (2 marks)

3. In the figure, PQR is a tangent to the circle centre O at Q. Angle $PQS = 28^o$, angle $UTQ = 54^o$ and UT = TQ.



Stating reasons, calculate the angles:

(a) ∠STQ (2 marks)

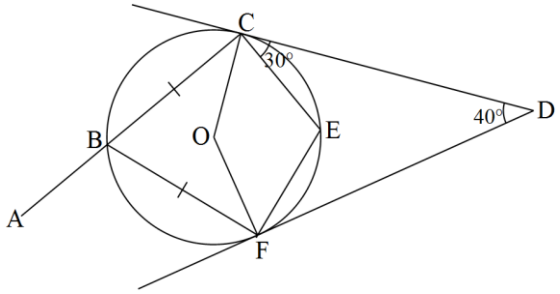
(b) ∠TQU (2 marks)

(c) ∠TQS (2 marks)

(d) Reflex ∠UOQ (2 marks)

(e) ∠TQR (2 marks)

4. In the figure below, O is the centre of the circle. CD and FD are tangents to the circle at C and F respectively. $\angle DCE = 30^o$ and $\angle CDF = 40^o$. ABC is a straight line and $BC = BF$.



Giving reasons, find:

(a) $\angle FCE$ (2 marks)

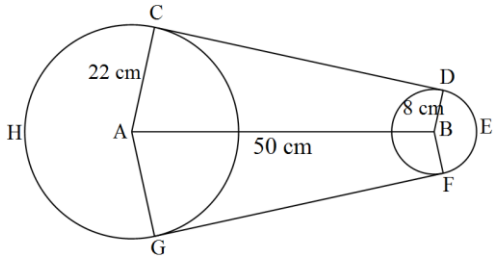
(b) $\angle CBF$ (2 marks)

(c) $\angle EFD$ (2 marks)

(d) $\angle BCO$ (2 marks)

(e) Reflex $\angle FOC$ (2 marks)

5. The figure below represents two pulley wheels, centres A and B with a string CDEFGHC tied round them. Radius of wheel centre A = 22 cm, radius of wheel centre B = 8 cm, AB = 50 cm, CD and GF are tangents to the circles.



Taking $\pi = 3.142$ where necessary, calculate:

(a) The length of CD. (2 marks)

(b) The obtuse angle CAG to 2 decimal places (2 marks)

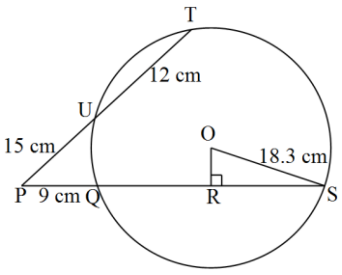
(c) The arc length:

(i) GHC correct to 2 decimal places. (2 marks)

(ii) DEF correct to 2 decimal places. (2 marks)

(d) The total length of the belt. (2 marks)

6. In the figure below, OS is the radius of the circle centre O. Chords SQ and TU are extended to meet at P and OR is perpendicular to QS at R. OS = 18.3 cm, PU = 15 cm, UT = 12 cm and PQ = 9 cm.



(a) Calculate the length of:

(i) QS (2 marks)

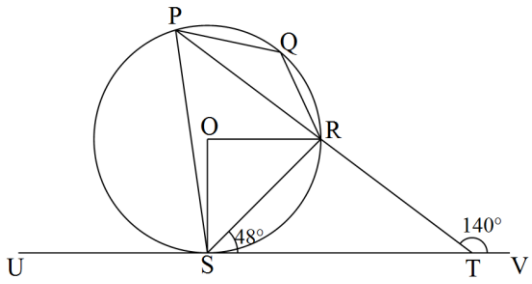
(ii) OR (2 marks)

(b) Calculate, correct to 1 decimal place:

(i) The size of angle ROS (3 marks)

(ii) The length of the minor arc QS (3 marks)

7. In the figure below, P, Q, R and S are points on the circle centre O. PRT and USTV are straight lines. Line UV is a tangent to the circle at S, $\angle RST = 48^o$ and $\angle RTV = 140^o$



(a) Calculate the size of:

(i) $\angle ORS$ (1 mark)

(ii) $\angle USP$ (1 mark)

(iii) $\angle PQR$ (1 mark)

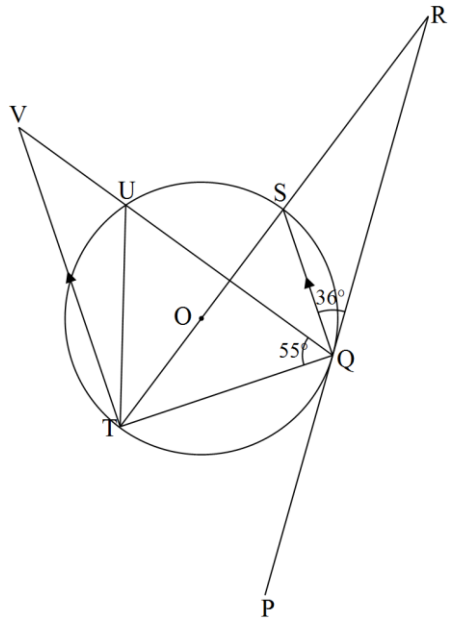
(iv) $\angle PSO$ (1 mark)

(b) Given that RT = 6 cm and ST = 9 cm, calculate:

(i) The length of line PR; (3 marks)

(ii) The radius of the circle to 3 s.f. (3 marks)

8. In the figure below, PQR is the tangent to the circle at Q. TS is a diameter and TSR and QUV are straight lines. QS is parallel to TV. Angle SQR = 36° and angle TQV = 55°.



Find the following angles, giving reasons for each answer.

(a) QTS (2 marks)

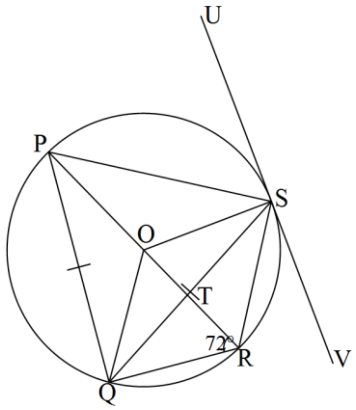
(b) QRS (2 marks)

(c) QVT (2 marks)

(d) UTV (2 marks)

(e) PQT (2 marks)

9. In the figure below, PR is a diameter of the circle centre O. Points P, Q, R and S are on the circumference of the circle. Angle PRQ = 72°, QS = QP and line USV is a tangent to the circle at S.



Find the following angles, giving reasons for each answer:

(a) QPR (2 marks)

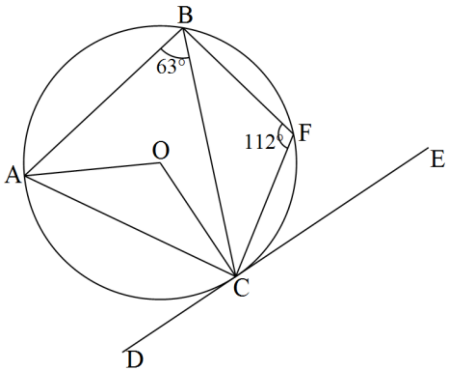
(b) PQS (2 marks)

(c) OQS (2 marks)

(d) RTS (2 marks)

(e) RSV (2 marks)

10. In the figure below, O is the centre of the circle. DCE is a tangent at C. $\angle ABC = 63^o$ and $\angle BFC = 112^o$.



Stating reasons, find:

(a) Angle ACD. (2 marks)
(b) Obtuse angle AOC. (2 marks)
(c) Angle BAC. (2 marks)
(d) Angle BCE. (2 marks)
(e) Angle OCB. (2 marks)

Answers

Intersecting Chords

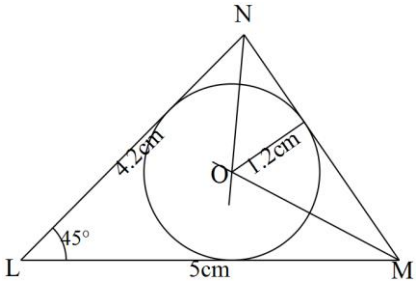
1. 9 cm
2. (i) SU = 16 cm (ii) QT = 12 cm
3. 12 cm
4. 11.8 cm
5. (a) 4 cm (b) 12 cm
6. 16 cm or 5 cm
7. 10 cm
8. 8 cm
9. (a) 5 cm (b) 19.6 cm
10. $4\sqrt{3}$ cm

Angle Properties of a Circle

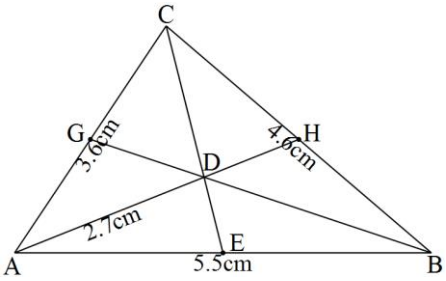
1. $\angle RST = 55^o$
2. (a) 76^o (b) 66^o
3. $\angle FBD = 60^o$, $\angle FDB = 50^o$, $\angle DBC = 70^o$
4. 56^o
5. (a) $\angle ADB = 70^o$ (b) 135^o
6. $\angle UPQ = 18^o$
7. $x = 75^o, y = 70^o, z = 50^o$
8. $\angle ABD = 51^o, \angle CBD = 31^o$
9. (a) 52^o (b) 48^o
10. 63^o

Construction

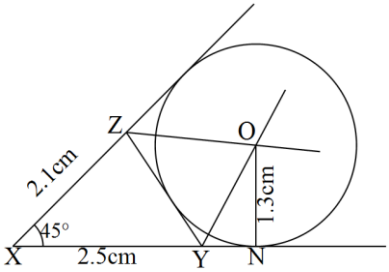
1. $r = 1.2cm$



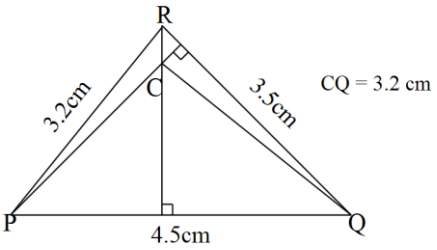
2. AD =2.7 cm



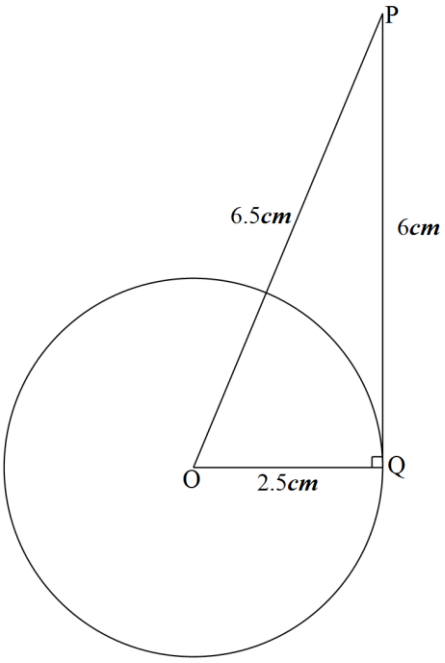
3. $r = 1.3cm$



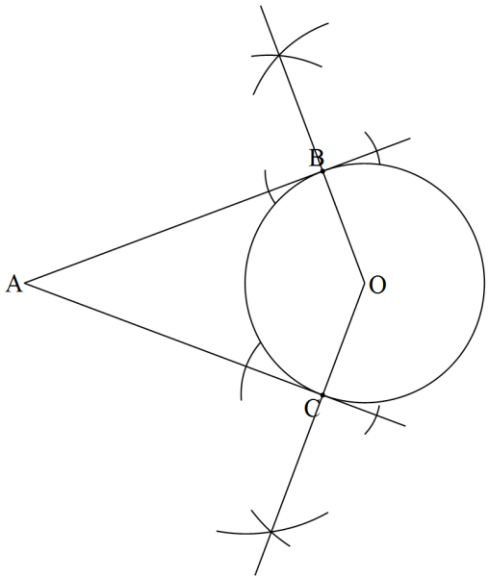
4. Orthocentre is where the altitudes intersect



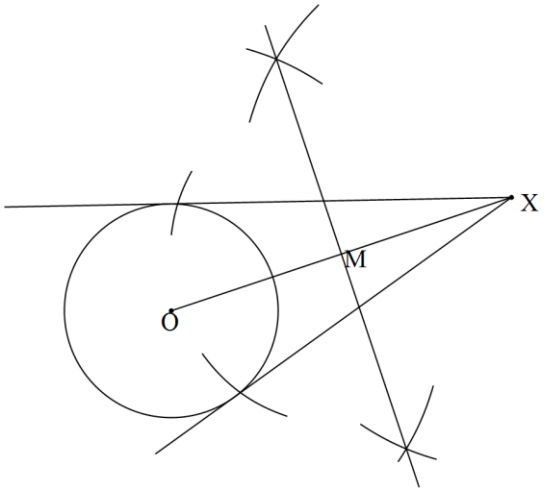
5. PQ = 6.5*cm*



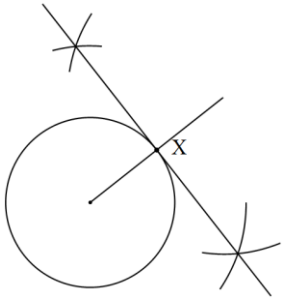
6.



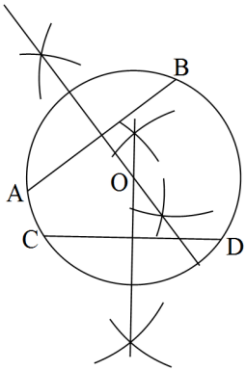
7.

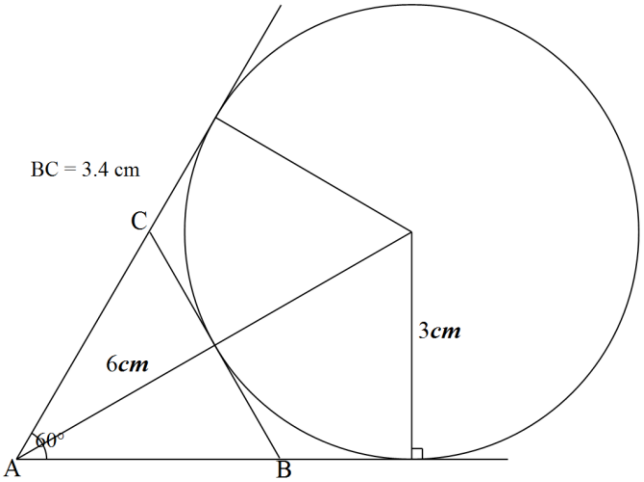


8.



9.





10. The line from A to the ex – centre measures 6 cm.

General Exercise

1. 24 cm
2. 19.66 cm
3. 12 cm
4. 4 cm
5. 24 cm
6. 213.7
7. 36 cm
8. 6 cm
9. 44.73
10. $4\sqrt{2}$

Section II Questions

1. (a) (i) 52^o (ii) 26^o (b) (i) 15.76 (ii) 8.54

2. (a) 30^o (b) 40^o (c) 50^o (d) 30^o (e) 130^o

3. (a) 28^o (b) 63^o (c) 35^o (d) 252^o (e) 117^o

4. (a) 40^o (b) 70^o (c) 40^o (d) 35^o (e) 220^o

5. (a) 48 cm (b) 147.48^o (c) (i) 81.61 (ii) 29.68 (d) 207.29

6. (a) (i) 36 cm (ii) 3.3 cm (b) (i) 79.6^o (ii) 50.9

7. (a) (i) 42^o (ii) 88^o (iii) 136^o (iv) 2^o (b) (i) 7.5 cm (ii) 5.40 cm

8. (a) 36^o (b) 18^o (c) 35^o (d) 19^o (e) 54^o

9. (a) 18^o (b) 36^o (c) 18^o (d) 54^o (e) 54^o

10. (a) 63^o (b) 126^o (c) 68^o (d) 68^o (e) 22^o